19. State and prove Hadamard's Factorization theorem.

20. Let $\Omega \subseteq C$ be a simply connected domain, and let $f \in H(\Omega)$ be such that $0 \notin f(\Omega)$ and $1 \notin f(\Omega)$. Then prove that there exists a $g \in H(\Omega)$ such that $f(z) = \frac{1+\cos(\pi \cos \pi g(z))}{2}$ and with the property that $g(\Omega)$ contains no open disk of radius 1.

Shanmuga Industries Arts & Science College
T.V.Malai

APRIL/MAY 2024

DMA41/GMA41 — COMPLEX ANALYSIS — II

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define Residue at a fixed point.

2. Find residue of $f(z) = \frac{(1+z^2)^{n+k}}{z^{2n+1}}$.

3. State Gauss Error integral.

4. What is called convergent improper integral?

5. Define Holomorphic extension.

6. Define singular point.

7. State Weierstrass Factorization theorem.

8. Define Zeta function.

9. Define open mapping.

10. State Hurwitz' theorem.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Evaluate $I_\alpha = \int_{|z|=1} \frac{Rez}{z-\alpha} dz, |\alpha| \neq 1$.

Or

(b) State and prove Rouche's theorem.

12. (a) Show that $\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2+1} dx = \frac{\pi(1-e^{-2})}{2}$.

Or

(b) For $\theta \in \left[0, \frac{\pi}{2}\right]$, show that $\sin\theta \geq \left(\frac{2}{\pi}\right)\theta$.

13. (a) If $f(z) = \sum_{n\geq 0} a_n z^n$ has a radius of convergence $R > 0$, then prove that f must have atleast one singularity on $|z| = R$.

Or

(b) State and prove Schwarz's Reflection principle for analytic functions.

14. (a) Prove that the series $\sum_{k=1}^{\infty} a_k$ converges absolutely iff the series $\sum_{k=1}^{\infty} Log(1+a_k)$ converges absolutely.

Or

(b) Prove that "For $p \in N \cup \{0\}$, we have $|1 - E_p(z)| \leq |z|^{p+1}$ for all $|z| \leq 1$".

15. (a) State and prove open mapping theorem.

Or

(b) State and prove Bloch's theorem.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Cauchy's Residue theorem.

17. Prove that $\int_0^{\infty} \frac{x^{m-1}}{1+x^n} dx = \frac{\pi}{n \sin\left(\frac{m\pi}{n}\right)}$, for $m, n \in N$ with $n > m > 0$.

18. State and prove Monodromy theorem.